# TIME-DEPENDENT SELF SIMILARITY IN ATMOSPHERIC TURBULENCE ACROSS DIURNAL AND TRANSITIONAL REGIMES

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**Abstract:** Atmospheric turbulence is traditionally characterized by stationary self – similar scaling, particularly within the inertial subrange, as described by classical turbulence theories. However, real atmospheric flows are inherently non-stationary, shaped by dynamic boundary conditions imposed by diurnal forcing, synoptic-scale transitions and reduce mesoscale convective processes. This study introduces a novel framework wherein the scaling exponents of turbulent structure functions evolve dynamically in time, capturing the transient nature of self – similarity in response to evolving meteorological conditions.

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## Introduction:

Turbulence in the atmosphere is one of the most complex and multiscale phenomena in fluid mechanics. While controlled laboratory turbulence can often be approximated as statistically stationary—meaning statistical properties do not change in time—real atmospheric flows are rarely so idealized. The Earth's surface, solar radiation, weather systems, and complex terrain all impose time-dependent forcing and boundary conditions.
The abstract provides three key points and the key points are discussed below:
**i.Classical View:** Turbulence scaling laws—such as **Kolmogorov's 1941** theory—assume statistical stationarity and self-similarity, particularly in the inertial subrange of scales.
**ii.Reality:** The actual atmosphere is non-stationary; both the mean flow and the turbulence statistics evolve over time.
**iii.Novel Contribution:** A dynamic scaling exponent framework that allows the exponents in structure functions to vary with time, reflecting transient self-similarity linked to changing meteorological conditions. Now we describe each points briefly in view of mathematical aspect.

## 1.Classical Turbulence Theory:

Classical turbulence theory emerged as an attempt to describe the chaotic, multi-scale nature of turbulent flows from the perspective of both mathematics and physics. Fundamentally, turbulence is governed by the Navier–Stokes equations for incompressible fluids, which embody conservation of mass and momentum. However, due to the nonlinear advection term $\mathbf{(u\cdot\nabla)\,u}$, exact analytical solutions are generally intractable at high **Reynolds numbers**.
In 1941, **A. N. Kolmogorov** formulated the cornerstone of classical turbulence theory, proposing that within the inertial subrange, turbulent energy cascades from large eddies to progressively smaller scales without loss, until viscous dissipation occurs at the Kolmogorov microscale. The velocity statistics in the inertial subrange depend only on the energy dissipation rate per unit mass, $\varepsilon$ and the wavenumber , k. Dimensional analysis then yields the famous -5/3 energy spectrum law:

$$E(k) \sim \varepsilon^{2/3} k^{-5/3}.$$

The theory assumes homogeneity, isotropy, and statistical stationarity. Kolmogorov's scaling predicts that the second-order velocity structure function follows

$$S_2(r) = \langle [u(x+r) - u(x)]^2 \rangle \sim (\varepsilon r)^{2/3}$$

where ε is the mean energy dissipation rate, x is a point in space (the spatial position vector) and r is the separation between two points a vector r. This power-law spectrum has been verified across numerous laboratory and atmospheric measurements.
Mathematically, turbulence is studied using statistical approaches such as Reynolds averaging and correlation functions. Physically, it represents a transfer of kinetic energy across scales, constrained by conservation laws but shaped by instability and chaotic interactions. Despite its success, classical theory faces limitations in non-stationary, anisotropic, and stratified flows, motivating modern extensions.

## 2.Atmospheric Turbulence and the Navier–Stokes Equations:

Atmospheric turbulence is mathematically described by the **Navier–Stokes equations**, which express the conservation of momentum in a fluid. For incompressible flow, the governing system consists of two parts:

### i.Continuity Equation

$$\nabla \cdot u = 0$$

where u (x, t) is the velocity field.

### ii.Momentum Equation (Navier–Stokes)

$$\partial u/\partial t + (u \cdot \nabla)\, u = -1/\rho \nabla p + \nu \nabla^2 u + f$$

where

**u** is the velocity field
**p** is the pressure
**ν** is the kinematic viscosity
**f** are external forces

In the atmosphere, this framework is applied within the planetary boundary layer, where turbulence strongly influences momentum, heat, and moisture transport. The nonlinearity of the advection term **(u·∇) u** generates eddies of multiple scales, leading to the well-known energy cascade described in turbulence theory. Although the **Navier–Stokes equations** fully describe turbulence, their complexity at high **Reynolds numbers** makes direct solutions infeasible. As a result, researchers rely on approximations such as **Reynolds-Averaged Navier–Stokes (RANS), Large Eddy Simulation (LES**), and statistical theories.

Thus, while atmosphere turbulence is fundamentally governed by the Navier–Stokes equations, practical understanding depends on modelling approaches that capture the essential physics of chaotic, multiscale motions.

**3.Cause of Incompressible Approximation in Atmosphere:**

Even though the atmosphere is compressible (air density changes with pressure/temperature), many turbulence studies assume **incompressibility** because at scales much smaller than the atmospheric scale height, density fluctuations are negligible compared to velocity/pressure variations.
Flows with low **Mach number (M≪1)** behave nearly incompressible.
Thus, turbulence in the planetary boundary layer (PBL) is often modelled with incompressible Navier–Stokes.

**i.Turbulence and Nonlinearity**

The critical part is the nonlinear advection term:

$$(u\cdot\nabla)\,u$$

This term allows energy transfer between scales- large eddies break into smaller eddies, forming the energy cascade described by **Kolmogorov's** turbulence theory.

**ii.Dimensionless Numbers**

Turbulence behavior is characterized by:

- Reynolds number: Re=UL/v where u = characteristic velocity L = characteristic length.
- v = kinetic viscosity of air (≈1.5x$10^{-5}$ $m^2/s$ )
- High Re in the atmosphere ($_{10}^{6}$- $_{10}^{10}$) → turbulence dominates over viscosity.

**Richardson number** (**Ri**): Ratio of buoyancy to shear effects, important in stratified turbulence (stably or unstably stratified atmosphere).

**iii.Modelling Atmospheric Turbulence**

Since exact solutions of Navier–Stokes in turbulence is impossible, we use approximations:

- **Direct Numerical Simulation (DNS):** Solves full Navier–Stokes → feasible only for low Reynolds number flows (not realistic for atmosphere).
- **Large Eddy Simulation (LES**): Resolves large eddies explicitly, parameterizes smaller ones.
- **Reynolds-Averaged Navier–Stokes (RANS**): Averages equations, introduces Reynolds stresses that need turbulence models **(e.g. k–ϵ, k –ω).**

**4. Atmospheric-Specific Forces:**

For the atmosphere, f includes:

- Coriolis force: from Earth's rotation, important at large scales.

- Buoyancy: arises from temperature differences (via **Boussinesq approximation**).
- Surface drag and heat fluxes: boundary-layer turbulence.

**5. Real Atmospheric Flows Vs Non-stationary:**

While this classical version of turbulence works remarkably well in lab flows or over short timescales, the real atmosphere is subject to temporal modulation of turbulence due to:
**i.Diurnal Forcing:** Solar heating causes the atmospheric boundary layer (ABL) to grow during the day (convective) and collapse at night (stable).
**ii.Synoptic-Scale Transitions:** Passage of weather fronts, pressure systems alter wind shear and stability.
**iii.Mesoscale Convective Systems:** Localized but powerful convective events rapidly change turbulence regimes.

These factors modulate:
- Mean wind speed
- Thermal stratification
- Turbulent Kinetic Energy (TKE) production
- Integral length scale L(t)
- Dissipation rate $\in(t)$

**6. Practical Implications:**

Dynamic scaling analysis is crucial for:
- Wind energy forecasting
- Pollutant dispersion modelling
- Aviation turbulence hazard assessment
- Climate model parameterization of turbulence

**7. Conclusion:**

From the present discussion we can say that the turbulence remains one of the most complex and fascinating phenomena in fluid dynamics, bridging physics and mathematics in the study of chaotic motion. At it's foundation, turbulence is governed by the **Navier–Stokes equations**, which fully describe incompressible fluid motion but are analytically intractable at the high Reynolds numbers typical of the atmosphere. This limitation has led to statistical and spectral approaches that reveal the universal properties of turbulent flows.
A cornerstone of this understanding is **Kolmogorov's 1941 theory** which introduced the concept of the **inertial subrange**. In this intermediate region, energy cascades from large to small scales without direct viscous influence, producing the celebrated **-5/3 law** for the energy spectrum and the $(\varepsilon r)^{2/3}$scaling of velocity structure functions. This universality highlights the scale invariant self-similar nature of turbulence across scales, despite the apparent randomness of the motion.

In the present work to make the model of atmospheric turbulence more realistic the concept of diurnal forcing, synoptic scale transitions and mesoscale convective systems have been introduced which are quite indispensable parameters for the understanding of atmospheric turbulence.

In conclusion, while atmospheric turbulence resists exact solutions, the combination of Navier–Stokes dynamics, Kolmogorov's scaling, and inclusion of diurnal forcing, synoptic scale transitions and mesoscale convective systems offer a powerful framework that continues to guide both theoretical advances and practical applications in case of atmospheric dynamics.

In future, work may be carried to implement proper mathematical inputs to introduce the phenomena of diurnal forcing, synoptic scale transitions and mesoscale convective systems.

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